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AUTHOR'S CLOSURE

NOVAK ZUBER

THE author appreciates the continuous interest which Professor Forster shows for the author's work; however, to use Professor Forster's own words, there are errors in his comments which the interested reader will be well advised to look out for.

1. Professor Forster is in error when he writes and applies equation (14) with $q_b = q$ to *both* saturated and subcooled pool boiling. Were this the case the author would certainly not have differentiated between saturated boiling (see Part 1) and subcooled boiling (see Part 2). And consequently, he would *not* have derived equations (17), (18) and (19), which describe bubble growth in *subcooled boiling*. It is fortunate that this misrepresentation was brought up, for if Dr. Forster misunderstands the derivations, further discussion is certainly required.

For saturated pool boiling at low heat flux densities (see Appendix C), it seemed that setting $q_b = q$ would be a good first approximation (see Table 3 and Fig. 3). On the other hand, in subcooled boiling the author was aware that $q_b \neq q$. *This is why the derivation from equations* (16) *to* (19) *was undertaken in the first place.* In subcooled boiling no assumption concerning the value of q_b was needed; the constant was evaluated from the condition that $\dot{R} = 0$ at $t = t_m$ which led to equation (16). By means of this equation q_b was related to a *measurable* quantity, i.e. to t_m, and thereby eliminated from equations (17), (18) and (19). Of course if one

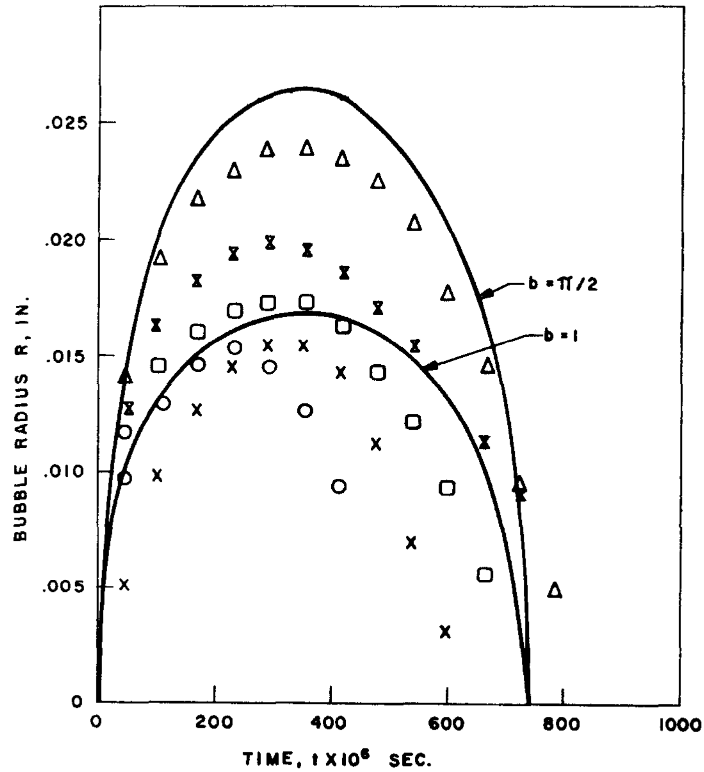

FIG. 11. Comparison of equations (17) and (22) with Ellion's experimental data for subcooled boiling; cf. Fig. 4.

employs, in subcooled boiling, the incorrect relation $q_b = q$ as Professor Forster did in making up his tables and figures, incorrect results will occur. It is unfortunate that Dr. Forster expended so much of his efforts along this line for when, in subcooled boiling, equation (17) is compared with experimental data the results shown on Figs. 11 and 12 are obtained. These curves were calculated by using the average experimental values for t_m ($t_m = 3{\cdot}7 \times 10^{-4}$ sec and $t_m = 3 \times 10^{-4}$ sec) and for $T_w - T_s$ ($\Delta T = 59$°F and $\Delta T = 62$°F). It is seen that both equations (17) and (18) (see Appendix D) predict results that are within the reproducibility of the experimental data.

Dr. Forster apparently believes that the author chose an equation of the form, $R = a\sqrt{t} - bt$, arbitrarily and was thereby able to obtain good results quite independent of physical considerations. The step from equation (16) to (19) was not merely "advantageous"—it was necessitated by the fact that in subcooled boiling $q_b \neq q$. And while Professor Forster has acquainted us with the properties of parabolas he has failed to note that not all, indeed very few, of these parabolas will have for the coefficients a and b values which predict results that are in agreement with experimental data as, for example, is the case with equations (17) and (18).

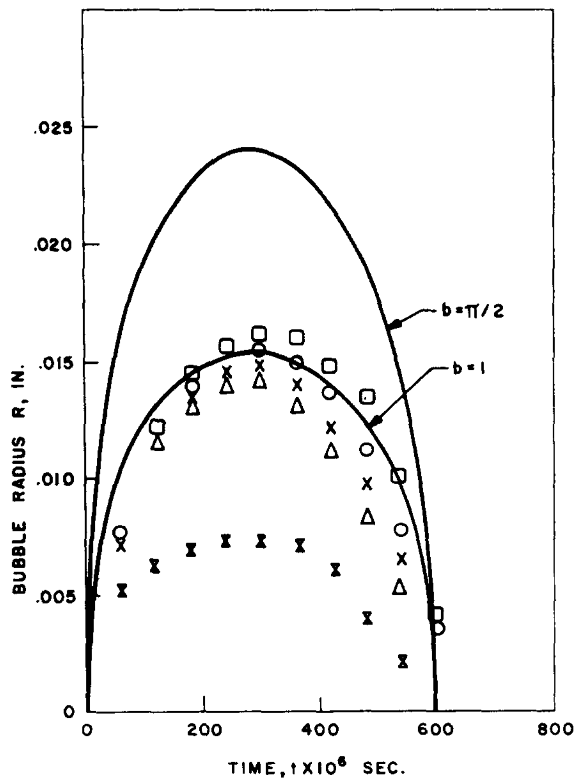

FIG. 12. Comparison of equations (17) and (22) with Ellion's experimental data for subcooled boiling; cf. Fig. 5.

Since Professor Forster's other arguments related to his first point are based on the same misinterpretation, i.e. that for subcooled boiling $q_b = q$, there is no point in discussing them further.

2. The efforts to which Dr. Forster went to check the accuracy of the author's statements indicate a zeal not often found in reviewers. It is unfortunate that he did not match his own efforts with a similar passion for accuracy. In Fig. 13 the author reproduces Fig. 39 from page 64 (which is cited by Dr. Forster) of Ellion's report (1954). The reader will note that the curve for degassed water has clearly marked upon it "Reference 15". Reference 15 in Ellion's report is the report by Gunther and Kreith. In other words, as the author stated, for the test points used (and tabulated in Appendix D) the temperature was taken from another report (i.e. Ellion used the data of Gunther and Kreith).*

The remaining paragraph of Dr. Forster's point number 2 actually refers to the misunderstanding which guided his comments in his point number 1.

3. For liquids at saturation temperature bubbles leave the heating surface while still growing, i.e. $\dot{R} > 0$, or sometimes under the condition that $\dot{R} \approx 0$. The departure is determined by the hydrodynamic conditions in the liquid which, as the reader will note from Appendix C, were not taken into account in the present analysis. This means that for liquids at

* We note that the values for degassed water shown on Ellion's figure are lower by approximately 12F° than the original data of Gunther and Kreith. This difference is due to an error in reproduction.

(Private communication from Dr. M. Ellion, National Science and Engineering Co., Pasadena, California, and from D. R. Bartz, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California.)

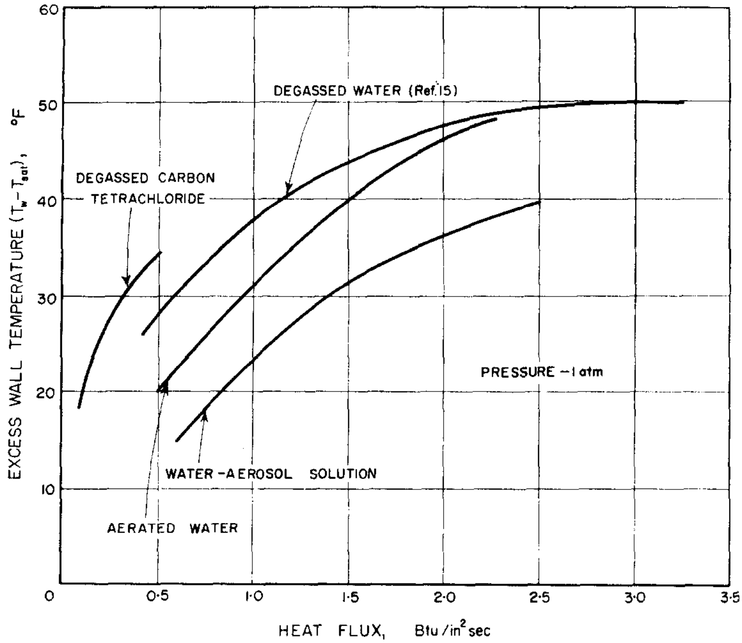

FIG. 13. Figure 39 reproduced from page 64 in Ellion's report (1954).

saturation, equations (13) and (14) should not be extended in the region $\dot{R} < 0$.

4. Professor Forster's fourth comment is a restatement of his misinterpretation in yet another form. It has been answered in the author's reply to his first point.

5. Professor Forster also states, "It is not quite reasonable to involve Bosnjakovic and the late Professor Jakob's names in this theory." The reader will note that the theory of Bosnjakovic and Jakob is discussed in Section 1.1.* It was pointed out, in Section 1.4, that subsequent, more detailed studies have substantially confirmed the energy considerations of Bosnjakovic and Jakob and the formulation of Fritz and Ende. The reader has only to compare equations (5) and (12) to verify this statement. A conceptual model is not modified, to any extent, by a multiplying constant of the order of unity, neither is it modified by the solution of the one dimensional heat conduction equation, i.e. of equation (2), for different initial conditions. It is for this reason that the author referred to the bubble growth or collapse process which is limited by the rate of heat transfer as the process which is described by the theory of Bosnjakovic and Jakob.† It is left to the reader to decide for himself whether it was "not quite reasonable" of the author to involve the names of Bosnjakovic and Jakob by giving credit to their fundamental contributions.

6. The author agrees with Professor Forster that the problem of bubble growth can be analyzed as an initial value problem as it was, indeed, formulated by Fritz and Ende in 1936. It was stated in the Introduction that this approach was adopted by Griffith (1956, 1958), Savic (1958), Bankoff and Mikesell (1958) and by Kaminski (1959). Such an approach involves two assumptions: first, one has to assume an initial temperature distribution, and, second, one has to assume, implicitly, that this initial temperature distribution will not be altered as the bubble grows and penetrates the turbulent region. Because of the high intensity of turbulent convection whether this second assumption is valid may be open to question; this point has already been brought up by Griffith and by Bankoff and Mikesell.

* The complete formulation of the problem is due to Plesset and Zwick and to Romie as noted in Section 1.4.

† Perhaps "the conceptual model" would have been a more appropriate expression than "theory"; this, however, is only a question of semantics.

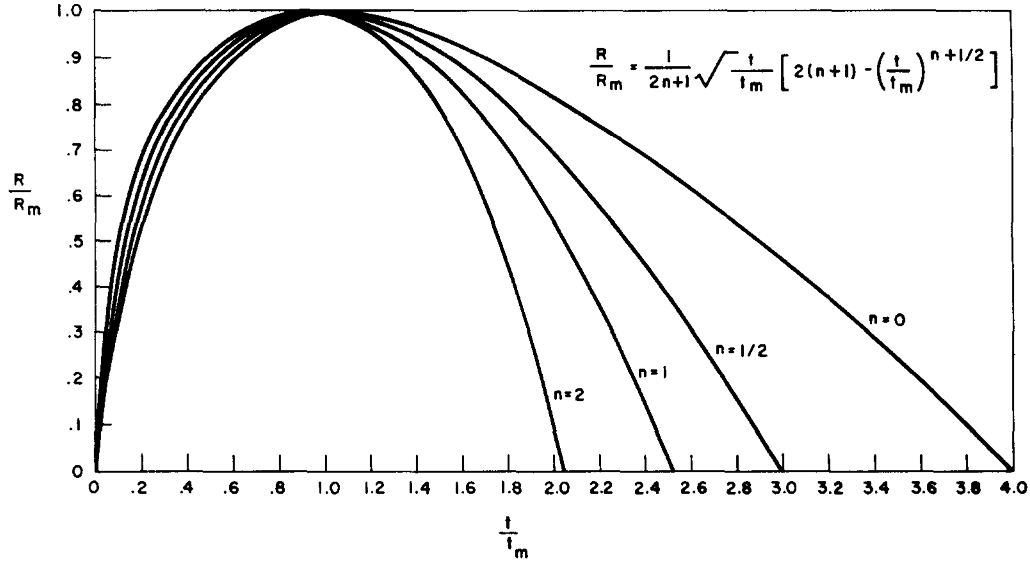

FIG. 14. Plot of equation (E5) showing the effect of a time dependent heat flux q_b.

Instead of solving an initial value problem (and making these two assumptions) the author made an energy balance, see equation (6), and assumed that the rate of heat transfer from the bubble interface to the bulk liquid, i.e. q_b, was constant. The way in which the heat flux q_b was evaluated was discussed in the paper as well as in Part 1 of this Closure. The justification of a constant heat flux density can be made in view of the hydrodynamic conditions, i.e. in view of the high turbulence. One could have assumed also that the effect of a bubble penetrating into the cold, turbulent liquid would be to produce a time dependent instead of a constant heat flux. Thus, in the general case, we can write instead of equation (6) the following energy balance

$$L\rho_v \frac{\mathrm{d}R}{\mathrm{d}t} = b\left[k\frac{T_w - T_s}{\sqrt{(\pi a t)}} - Ct^n\right]. \quad \text{(E1)}$$

As before we determine the value of the constant C from the condition that $\dot{R} = 0$ when $t = t_m$. It follows then that radius vs. time relation is given by

$$R = b\frac{2}{\pi}\frac{\Delta T c \rho_L}{L\rho_v}\sqrt{(\pi a t)}\frac{1}{2(n+1)} \left[2(n+1) - \left(\frac{t}{t_m}\right)^{n+1/2}\right]. \quad \text{(E2)}$$

The time t_c for the bubble to collapse is

$$\frac{t_c}{t_m} = [2(n+1)]^{2/2n+1}. \quad \text{(E3)}$$

The maximum bubble radius becomes

$$R_m = \frac{b}{\pi}\frac{\Delta T c \rho_L}{L\rho_v}\sqrt{(\pi a t_m)}\frac{2n+1}{2(n+1)}. \quad \text{(E4)}$$

It follows from equation (E4) and (E2) that the dimensionless equation in terms of the normalized co-ordinates becomes

$$\frac{R}{R_m} = \frac{1}{2n+1}\sqrt{\frac{t}{t_m}}\left[2(n+1) - \left(\frac{t}{t_m}\right)^{n+1/2}\right]. \quad \text{(E5)}$$

It can be seen that when the exponent n is zero, equations (E1), (E2), (E4), and (E5) reduce to equations (13), (17), (18) and (19) respectively. Equation (E5) is plotted on Fig. 14 for various values of n. It can be seen that the growth is practically unaffected by the choice of n. By comparing these curves with the experimental data plotted on Figs. 6 and 7, it can be seen that equation (E5) describes both the bubble growth and collapse.

In summary, the error in Professor Forster's comments is his misinterpretation that in subcooled boiling $q_b = q$, i.e. that equation (14) with $q_b = q$ applies to subcooled boiling. As indicated in Part 2 of the text, the growth of a bubble in subcooled boiling is given by equations (17), (18) and (19). The comparison of predicted values with experimental data is shown on Table 3, Figs. 3, 6, 7, 11, 12, 14 and in Appendix D on Tables A1, A2 and A3. No further comment appears necessary.